The use of Visual Recurrence Analysis and Hurst exponents as qualitative tools for analysing financial time series

Branko Pecar, PhD

University of Gloucestershire
bpecar@glos.ac.uk

Abstract. Complex dynamical systems existing in an unobservable multidimensional space can be successfully represented via an artificial vector space, created from the single output variable of such a dynamical system. Visual Recurrence Analysis (VRA) method enables visualisation of the single time series in such a space. This paper uses VRA method and Hurst exponents as the means of estimating, describing and qualifying some of the properties of several financial time series. Weak evidence of chaos in daily returns is indicated.

Keywords. Visual recurrence analysis, Hurst exponent, time series analysis, embedded time delay vector space, nonlinear analysis of financial data

1. Introduction

Many economic and financial phenomena are represented by highly complicated and complex variables defined by numerous factors. Most of these complex phenomena are symbolized by a single scalar time series, representing the output of this complex system. Using certain techniques it is possible, from a single output time series, to reconstruct a representation of an unobservable space, as well as the underlying attractor embedded in such a space. This expansion of a time series into a higher-dimensional space is achieved by delayed coordinate embedding. A phase space portrait created in such a way is equivalent to the multidimensional system.

2. Creating a phase space

In order to create a phase space, observations from the time series need to be turned into vectors Y_t. This is easily achieved in the following fashion:

$$Y_t = (x_t, x_{t+\tau}, x_{t+2\tau}, \ldots, x_{t+(m-1)\tau}) \qquad (1)$$

Where,
x_t = time series observations; t = units of time; τ = time delay (sampling time); m = embedding dimension

A sufficient number of vectors Y_t can create a phase space that is topologically equivalent to the original and unobservable multidimensional space that hosts the underlying system. Once the system has been reconstructed it is possible to analyse the properties of such systems, otherwise hidden in the time window.

3. Visual Recurrence Analysis (VRA)

Recurrence analysis, as a method for identifying hidden patterns, was introduced by Eckmann, Kampshort and Ruelle [1]. A recurrence plot compares two vectors and when the same vector is compared to itself it produces a zero distance. Characteristically, because of this reason, all recurrence plots have a pronounced line of identity, i.e. the diagonal line. The more deterministic the time series is, the more regular the recurrence plot will be. The regularity is expressed through either diagonal lines or through some pattern formation, depending on the use of colours. To illustrate the above points we show recurrence plots for four characteristic time series, and they are: a straight line, a sinusoid, Gaussian white noise and a Wiener process (Brownian motion with Markov characteristics). Their plots are shown in Figs. 1 to 4.

Recurrence plots in Figs. 1 to 2 (and all the succeeding ones) were created using the 4.2 release of VRA software[1]. The colour scheme used in this paper was consistent. Fig. 5 provides an illustration of the colour scheme and the distances.

[1] The package was developed by Eugene Kononov and is available from the URL <http://home.netcom.com/~eugenek/download.html>

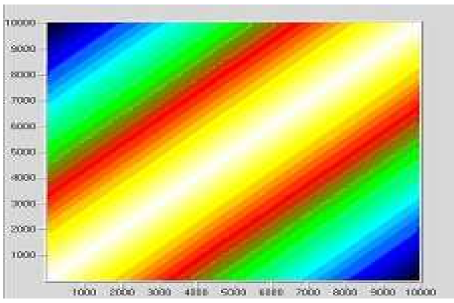

Fig. 1 Colour recurrence plot for a straight line

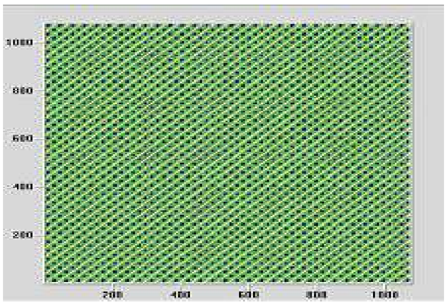

Fig. 2 Colour recurrence plot for a sinusoid

Recurrence plot analysis involves looking for lines parallel with the line of identity. However, it is very likely that only short parallel lines will be present. In general, the presence of short diagonal segments that mirror the line of identity indicate the presence of the so-called unstable periodic orbits embedded in the chaotic attractor of a deterministic system [2].

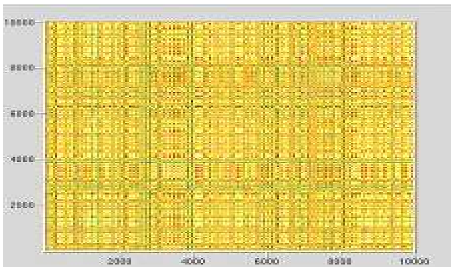

Fig. 3 Colour recurrence plot for Gaussian white noise

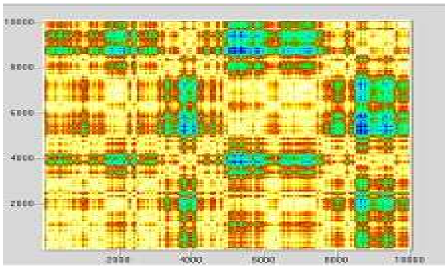

Fig. 4 Colour recurrence plot for the Wiener process (Brownian motion)

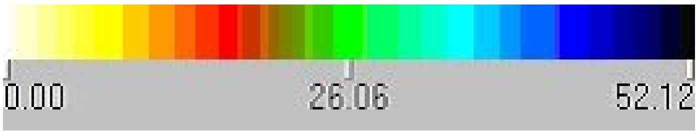

Fig. 5 Colour scheme used for recurrence plots

4. Analysing financial data

In the world of finance it is customary to analyse stock returns, rather than closing values of stocks. There are a variety of reasons for this, some of which are purely of a general economic nature (price scales are not proportional due to inflation, etc.), some are of a business nature (traders are interested in returns, rather than actual stock prices) and the others are more technical[2], as illustrated by a number of sources [3] [4]. Let us define stock price returns.

If a stock price at any point in time is defined as X_t and Δt is a change in time, then the price change is calculated as:

$$Z_t = X_{t+\Delta t} - X_t \qquad (2)$$

It can be inferred[3] that $\frac{Z_t}{X_t} \approx \log X_{t+\Delta t} - \log X_t$, so for the sake of convenience, the returns in this paper are defined as:

$$R_t = \log X_{t+\Delta t} - \log X_t \qquad (3)$$

[2] Returns have been successfully modelled by ARCH and GARCH models, whilst the actual stock values are often nonstationary and, therefore, difficult to handle by conventional analytical methods.

[3] Just as an indication take that (x-y)/y = (x/y)-1 ≈ ln (x/y)

For the purpose of examination in this paper, we have selected BP daily and minute closing value of shares (as quoted on NYSE). Both series contain 5,458 observations. The first series contains daily closing values of BP stock prices from 28 September 1981 until 9 May 2003. The second one contains BP minute closing stock values from 22 April 2003 at 09.30 until 9 May 2003 16.00.

A number of studies, arguably with more or less success, attempted to find the presence of low-dimensional chaos in stock data [5]. However very few studies attempted to do comparative analysis for different data frequencies of the same data set. This paper will address this problem by using two data sets of the same length, but each referring to a different time window (days and minutes).

5. Visual inspection of recurrence plots

As some of the stock value graphs exhibit nonstationarity, the series need to be differentiated first. After transforming the data, the recurrence plots for all four series were produced. Some of the recurrence plots indicated that perhaps some deeper pattern might be embedded in daily data.

The Hurst exponent [6] is an additional tool we used in this qualitative analysis. We calculated Hurst exponents for all four series and Fig. 6 contains the summary statistics. It is indicative that the Hurst exponent for daily differenced data shows that this data set is about 10% more "violent" than the minute differenced data set (the inspiration for this wording comes from Mandelbrot [7][4]).

	Daily values	Minute values	Differenced daily values	Differenced minute values
Hurst exponent	1.0127	1.0022	0.552	0.5074
Measurement error	±0.0976	±0.0812	±0.1835	±0.1738

Fig 6. Some of the properties of the analysed time series and their transformations

As indicated earlier, the next step in VRA analysis is to estimate the correct embedding dimension and the time delay. Expectation is that this extraction will yield a recurrence plot that provides a better indication about a possible underlying system that drives these results. Unfortunately in our case, no obvious pattern emerged. The absence of any diagonal lines parallel with the line of identity suggests that both daily and minute closing values are more likely to be results of some stochastic, rather than deterministic system.

There is a possibility that the two data series are so polluted with noise that no transformation can reveal any pattern. However, rather than removing the noise from the data, as recommended by some authors [8], we decided to keep the original data intact. Visual Recurrence Analysis (VRA) method as a qualitative method is fairly robust and insensitive to noise to the same degree as some of the analytical methods. Given this, the analysis of returns, rather than the actual stock values is possibly a better option.

6. Returns analysis and process memory

To start with, a surrogate data set for both the daily and minute returns was created. Rather than conducting statistical tests, in accordance with the spirit of this paper, the values of Hurst exponents for the original and surrogate data sets were estimated and they are displayed in Fig. 7.

	Daily returns	Surrogate daily returns	Minute returns	Surrogate minute returns
Hurst exponent	0.45579	0.59496	0.50852	0.51874
Measurement error	±0.12562	±0.11682	±0.17299	±0.10623

Fig. 7 Hurst exponent for the actual and surrogate data of daily and minute returns

[4] Mandelbrot states: "… fractional noises with a high value of H are the most violently fluctuating among fractional noises."

These values indicate that the returns for both data sets exhibit more random characteristics than would be expected from a data set that follows strictly Gaussian distribution. Actual daily returns are somewhat negatively autocorrelated, i.e. every positive move is more likely to be followed by a negative move than is the case with surrogate data. In the case of minute data, the differences are not so pronounced. These values indicate that daily data might be driven by a chaotic process, rather than a stochastic one. Following this hint, the recurrence plots for daily and minute returns were produced.

When we zoomed in on the daily return plot, the presence of short segments parallel with the line of unity, indicating the possible presence of unstable periodic orbits, started to emerge (Fig. 8). In Fig. 9 the colour scheme was reduced to make these lines more visible. What kind of deterministic process is driving these returns is impossible to tell using the given tools from this paper, but indications are that this system is chaotic, rather than linear stochastic.

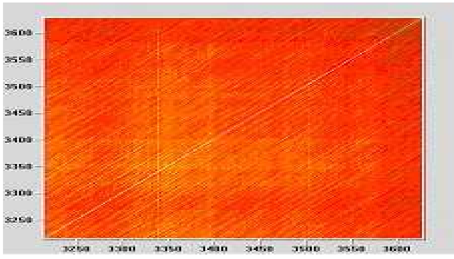

Fig. 8 Zoom into a region

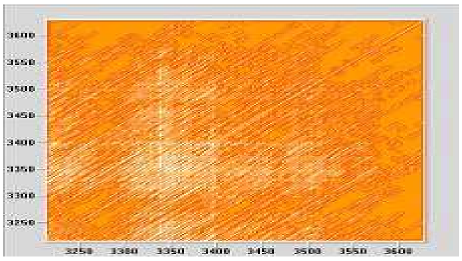

Fig. 9 Same as Fig. 8, but reduced colours

The question of process memory in returns is yet another issue that might help us identifying deeper meaning of the data available. In other words the question is: are the returns, separated by some amount of time, in any way related? We approached this problem in the following way. We measured the Hurst exponent for both daily and minute returns over a number of time delays[5]. What transpired was that most of the short returns have a Hurst exponent of approximately 0.5, which grows towards 1.0 as we increase time delays[6]. This indicates that returns tomorrow, when compared to todays, follow a white noise process, whilst returns that are sufficiently removed from today's returns have memory resembling an integrated white noise process, i.e. fractional Brownian motion. The only difference between daily returns and minute returns is that daily returns show more randomness than minute returns. Fig. 10 shows how the Hurst exponents change in accordance with the changes of time measuring different returns.

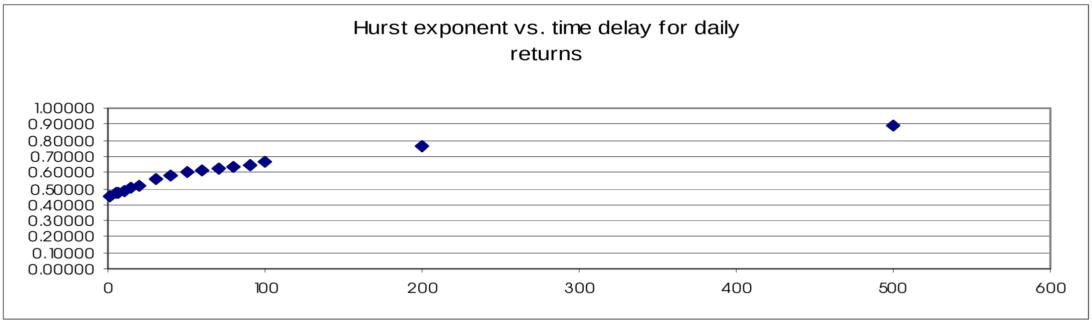

Fig. 10 Changes in Hurst value as the time delay for daily returns changes

The move for different time delays from, what appears to be a Gaussian distribution, towards some sort of leptokurtic distribution with fat tails has been known for quite some time [9]. The changes in Hurst exponent confirm this, although as expected, do not provide an answer as to what process drives these changes. This question exceeds the capabilities of the tools and qualitative analysis approach used in this paper.

---

[5] More specifically, returns were measured for 1,2,5,6,10,15,20-100, 200 and 500 time delays.

[6] In the case of daily returns H=0.897 for 500 delays and in the case of minute returns H=0.929 for 500 delays.

7. Conclusion

In this paper a qualitative approach was adopted to answer several questions about the characteristics of financial time series. Only one variable was examined (BP daily and minute stock movements) and only VRA analysis, coupled with Hurst exponent indicators, was used to achieve this. The paper attempted to address the issue of whether different sampling of financial time series could reveal the presence of low dimensional chaos and whether price returns could reveal the properties not so visible from the stock price closing values. First of all, the sampling interval (at least for the data sets used in this paper) made no difference. A need for future studies in which the same interval is sampled at different frequencies (sample rates) is more than evident. The second point is that the analysis indicated that both the daily and minute closing values are more likely to be stochastic than deterministic. In addition to this, daily closing data show some 10% more process memory than the minute data.

As far as the price returns are concerned, the VRA analysis indicates that this process might not be a random walk, but a manifestation of some hidden chaotic system. How this system could be characterised and what makes the transition of short-term return white noise into a long-term return chaotic system needs to be investigated using different tools. The findings also encourage further, more rigorous and quantitative analysis, particularly in the domain of embedded time delay coordinates.

References

1. Eckman, J.A., S.O. Kampshort, and D. Ruelle, *Recurrence Plots of Dynamical Systems.* Europhysics Letters, 1987. **4**: p. 973-977.
2. Holyst, J.A., M. Zebrowska, and K. Urbanowicz, *Observations of Deterministic Chaos in Financial Time Series by Recurrence Plots, Can One Control Chaotic Economy?* The European Physical Journal B, 2001. **20**: p. 531-535.
3. Meade, N., *A Comparison of the Accuracy of Short Term Foreign Exchange Forecasting Methods.* International Journal of Forecasting, 2002. **18**: p. 67-83.
4. Koopman, S.J. and C.S. Bos, *Time series models with a common stochastic variance for analysing economic time series*. 2002, http://www.tinbergen.nl/discussionpapers/02113.pdf.
5. Harrison, R.G., et al., *Non-linear Noise Reduction and Detecting Chaos: Some evidence from the S&P Composite Price Index.* Mathematics and Computer in Simulation, 1999. **48**: p. 497-502.
6. Hurst, H.E., *Long-term storage capacity of reservoirs.* Transactions of the American Societ of Civil Engineers, 1951. **116**: p. 770-808.
7. Mandelbrot, B.B., *Gaussian Self-Affinity and Fractals*. 2002, New York: Springer Verlag. 654.
8. Matassini, L., et al., *Optimizing of Recurrence Plots for Noise Reduction.* Physical Review E, 2002. **65**(021102).
9. Mantegna, R.N. and H.E. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance*. 2001, Cambridge: Cambridge University Press. 148.